

QUARK MATTER STRUCTURE IN NEUTRON STARS

H. HEISELBERG *

NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

ABSTRACT

For physically reasonable bulk and surface properties, quark matter has recently been found to coexist with nuclear matter inside neutron stars in a uniform background of electrons. The microstructure of this mixed phase starts out with a few quark matter droplets embedded in the nuclear matter but as the density of droplets increase towards the center of the neutron star they merge into rods, plates and other structures. The topology, size as well as Coulomb and surface energies of these structures depend sensitively on the quark/nuclear matter interface tension. A major fraction of the interior of neutron stars could consist of matter in this new phase, which would provide new mechanisms for glitches, cooling and supernovae.

1. The Structure of Quark Matter in Neutron Stars

Over the past two decades many authors¹ have considered the existence of quark matter in neutron stars. Assuming a first order phase transition one has, depending on the equation of states, found either complete strange quark matter stars or neutron stars with a core of quark matter surrounded by a mantle of nuclear matter and a crust on top. Recently, the possibility of a mixed phase of quark and nuclear matter was considered² and found to be energetically favorable. Including surface and Coulomb energies this mixed phase was still found to be favored for reasonable bulk and interface properties³. The structure of the mixed phase of quark matter embedded in nuclear matter with a uniform background of electrons was studied and resembles that in the neutron drip region in the crust. The resulting picture of a neutron stars is shown in Fig. 1. Starting from the outside, the crust consists of the outer layer, which is a dense solid of neutron rich nuclei, and the inner layer in which neutrons have dripped and form a neutron gas coexisting with the nuclei. The structure of the latter mixed phase has recently been calculated in detail⁴ and is found to exhibit rod-, plate- and bubble-like structures. At nuclear saturation density $n_0 \simeq 0.16\text{fm}^{-3}$ there is only one phase of uniform nuclear matter consisting of mainly neutrons, a small fraction of protons and the same amount of electrons to achieve charge neutrality. A mixed phase of quark matter (QM) and nuclear matter (NM) appear already around a few times nuclear saturation density - lower than the phase transition in hybrid stars. In the beginning only few droplets of quark matter appear but at higher densities their number increase and they merge into: QM rods, QM plates, NM rods, NM bubbles, and finally pure QM at very high densities if the neutron stars have not become unstable towards gravitational collapse.

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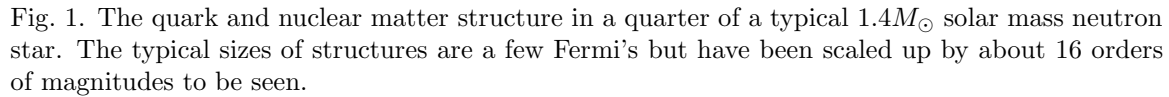


Fig. 1. The quark and nuclear matter structure in a quarter of a typical $1.4M_{\odot}$ solar mass neutron star. The typical sizes of structures are a few Fermi's but have been scaled up by about 16 orders of magnitudes to be seen.

2. The Phase Transition

The phase transition is determined by the phase coexistence conditions and the equations of states (EoS) of both NM and QM at essentially zero temperature. Whether a simple quadratic form³ for the NM EoS or more elaborate EoS based on Urbana/Argonne potentials, the results only varied quantitatively for a wide range of parameters (e.g., compressibility, symmetry energy, bag constant, α_s , m_s ,...). Qualitatively the same picture emerges: *The mixed phase is energetically favored and starts already around twice nuclear saturation density.*

Here we take for comparison another nuclear EoS, the Walecka relativistic mean field model⁵. Reproducing the nuclear saturation density and binding energy determines the scalar $g_S^2/m_S^2 = 2300 \text{ MeV/fm}^3$ and vector $g_V^2/m_V^2 = 1700 \text{ MeV/fm}^3$

couplings. For quark matter we assume the bag model equation of state

$$\epsilon_{QM} = (1 - \frac{2\alpha_s}{\pi}) \left(\sum_{q=u,d,s} \frac{3\mu_q^4}{4\pi^2} \right) + B + \frac{\mu_e^4}{12\pi^2}, \quad (1)$$

with the qcd fine structure constant $\alpha_s \simeq 0.4$ and bag constant $B \simeq 120 \text{ MeV/fm}^3$. We have taken all quark masses to be zero.

The energy densities of NM and QM are shown in Fig. 2 with two types of phase transitions depending on the phase coexistence conditions as will now be discussed.

2.1. Hybrid Stars

In hybrid stars it is assumed that each of the two phases are electrically neutral separately. Thus the proton density is equal to the electron density in nuclear matter and is given by β -equilibrium $\mu_n = \mu_p + \mu_e^{NM}$ at a given density. Likewise in the quark matter β -equilibrium $\mu_d = \mu_s = \mu_u + \mu_e^{QM}$ and charge neutrality determines all quantities for a given density.

Gibb's conditions $P_{NM} = P_{QM}$ and $\mu_n^{NM} = \mu_n^{QM}$ (the temperatures are vanishing in both phases) then determine a unique density at which the two bulk neutral phases coexist. This is the standard Maxwell construction and is seen in Fig. 2 as the double-tangent. In a gravitational field the denser phase (QM) will sink to the center whereas the lighter phase (NM) will float on top as a mantle. At the phase transition there is a sharp density discontinuity and generally $\mu_e^{NM} \neq \mu_e^{QM}$ so that the electron densities $n_e = \mu_e^3/3\pi^2$ are *different* in the two phases. This assumes that the sizes of QM structures are larger than electron screening lengths which, as discussed in ³, is *not* the case.

For very low bag constants the phase transition occur at densities lower than n_0 and the whole neutron star is a quark star except possibly for a hadronic crust⁷.

2.2. Mixed phase

We now consider the case where bulk QM is embedded in nuclear matter and the sizes of the QM structures are smaller than typical electron screening lengths. Thus the electron background is almost uniform which gives the extra condition $\mu_e^{NM} = \mu_e^{QM}$. On the other hand charge neutrality is only required on average and not in each phase. As a consequence QM and NM can coexist in a wide range of densities and as seen in Fig. 2(a) the energy density is always lower than the two bulk neutral phases when surface and Coulomb energies are neglected ($\sigma = 0$). We also observe that QM in the form of droplets appear at a lower density than the phase transition between the two bulk neutral phases with increasing density or pressure more and more NM is deconfined into QM. Defining the *filling fraction* $f = V_{QM}/(V_{NM} + V_{QM})$ as the fraction of the volume in the QM phase, we see that f increase continuously from zero to unity as the density increase from $\sim 2n_0$ to $\sim 9n_0$.

Fig. 2. (a) The full line gives the energy density of the droplet phase without surface and Coulomb energies ($\sigma = 0$). Also shown are the energy densities of electrically neutral bulk nuclear matter, quark matter in β -equilibrium, and the double tangent construction (dashed line) corresponding to the coexistence of bulk electrically neutral phases. (b) Energy densities of the droplet phase relative to its value for $\sigma=0$ for $\sigma = 10, 20$, and 30 MeV/fm^2 . When the energy density of the droplet phase falls within the hatched area it is energetically favored.

3. Why Is the Mixed Phase Energetically Favored?

This becomes evident by looking at charge densities. Consider quark matter immersed in a uniform background of electrons. Beta equilibrium insures that $\mu_d = \mu_s = \mu_u + \mu_e$, and therefore in the absence of quark-quark interactions, one finds the total electric charge density in the quark matter phase is given for $\mu_e \ll \mu_u \sim \mu_d \equiv \mu_q$ and $m_s \ll \mu_q$ by

$$\rho_Q = \frac{e}{3}(2n_u - n_d - n_s - 3n_e) \simeq \frac{e}{\pi^2} \left(\frac{1}{2}m_s^2\mu_q - 2\mu_e\mu_q^2 \right), \quad (2)$$

since $n_i = (\mu_i^2 - m_i^2)^{3/2}/\pi^2$. Assuming $m_s \simeq 150 \text{ MeV}$ and $\mu_q \simeq m_N/3$ the second term dominates except for small μ_e and so the droplet is negatively charged. The electron chemical potential in neutron stars depends strongly on the model for the

nuclear equation of state, but generally one finds $\mu_e \lesssim 170$ MeV and so $\rho_Q \lesssim -0.4e$ fm⁻³. Due to the high quark density, ρ_N is small compared with ρ_Q in Eq. (5) when quark matter occupies a small fraction of the volume.

The QM is therefore *negatively* charged and by immersing QM in the positively charged NM we can either remove some of the electrons from the top of the Fermi levels with energy μ_e , or we can increase the proton fraction in NM by which the symmetry energy is lowered. In equilibrium a combination of both will occur and in both cases *bulk energy is saved* and a lower energy density is achieved as seen in Fig. 2. However, we still have to pay the Coulomb and surface energies of the structures, as will now be estimated, and see whether the mixed phase is really energetically favored.

4. Properties of the Mixed Phase

4.1. Coulomb and Surface Energies of Structures

Surface and Coulomb energies determine the topology and length scales of the structures. Denoting the dimensionality of the structures by d ($d = 3$ for droplets and bubbles, $d = 2$ for rods and $d = 1$ for plates) the surface and Coulomb energies are generally⁴

$$\mathcal{E}_S = d\sigma \frac{4\pi}{3} R^2, \quad (3)$$

$$\mathcal{E}_C = \frac{8\pi^2}{3(d+2)} (\rho_Q - \rho_N)^2 R^5 \left[\frac{2}{d-2} \left(1 - \frac{d}{2} f^{1-2/d} \right) + f \right], \quad (4)$$

where σ is the surface tension, R the size of the structure, and ρ_Q and ρ_N are the total charge densities in bulk QM and NM respectively. For droplets ($f \simeq 0$) or bubbles ($f \simeq 1$) $d = 3$ and the Coulomb energies reduce to the usual term $\mathcal{E}_C = (3/5)Z^2e^2/R$ where Z is the excess charge of the droplet compared with the surrounding medium, $Ze = (4\pi/3)(\rho_Q - \rho_N)R^3$. Minimizing the energy density with respect to R we obtain the usual result that $\mathcal{E}_S = 2\mathcal{E}_C^\dagger$. Minimizing with respect to the continuous dimensionality as well thus determines both R and d .

We now estimate surface and Coulomb energies. When quark matter occupies a small fraction of space, $f \simeq 0$, one can show that the difference in energy between the droplet phase and bulk neutral nuclear matter varies as f^2 . In contrast to this, the contributions to the energy density from surface and Coulomb energies are linear in f . (See Eq. (6)) Similar results apply for f close to unity. This shows that the transitions to the droplet phase must occur via a first-order transition. However, if the surface and Coulomb energies are sufficiently large, the droplet phase may never be favorable. The energy-density difference between the droplet phase, neglecting surface and Coulomb effects, and two coexisting neutral phases is a few MeV/fm³, as may be seen from Fig. 2. This is very small compared with characteristic energy densities which are of order 1000 MeV/fm³. In Fig. 2(b) we show the energy density

[†]The condition for fission instability is contrarily: $2\mathcal{E}_S \leq \mathcal{E}_C$.

of the droplet phase for various values of the surface tension, relative to the value for $\sigma = 0$. For the droplet phase to be favorable, its energy density must lie below those of nuclear matter, quark matter, and coexisting electrically neutral phases of nuclear and quark matter. That is the droplet phase will be favored if its energy lies within the hatched region in Fig. 2(a+b). We see that whether or not the droplet phase is energetically favorable depends crucially on properties of quark matter and nuclear matter. For our model the droplet phase is energetically favorable at some densities provided $\sigma \lesssim 20$ MeV/fm². For comparison, using a quadratic Eos for NM³ one finds instead the more favorable condition $\sigma \lesssim 70$ MeV/fm². Given the large uncertainties in estimates of bulk and surface properties one cannot at present claim that the droplet phase is definitely favored energetically.

In these analyses several restrictions were made: the interfaces were sharp, the charge densities constant in both NM and QM and the background electron density was also assumed constant. Relaxing these restrictions generally allow the system to minimize its energy further. Constant charge densities may be a good approximation when screening lengths are much larger than spatial length scales of structures but since they are only slightly larger³ the system may save significant energy by rearranging the charges.

4.2. The Quark and Nuclear Matter Interface Tension

The surface tension is a crucial but unfortunately a poorly determined parameter (see Ref. ³ for a discussion). A rough estimate of the surface tension is the bag constant, B , times a typical hadronic length scale ~ 1 fm⁸ and gives $\sigma \simeq 50 - 450$ MeV/fm². Estimates from the MIT bag model and from lattice gauge calculations are somewhat lower. The surface tension may also depend on the densities on both sides of the interface. Whereas the densities inside the neutron star vary from nuclear matter density to about an order of magnitude larger, we have checked by detailed computation with several equation of states that the density difference over the surface does not vary by much. We therefore keep the surface tension as an unknown but density independent parameter.

4.3. Droplet Radii, Charge and Mass Numbers

When $f \sim 0$ spherical QM droplets form in NM whereas when $f \sim 1$ bubbles of NM are embedded in QM. The surface energy and Coulomb energies are given by Eqs. (3) and (4) for $f = 0$ or $f = 1$ and the minimization condition $\mathcal{E}_S = 2\mathcal{E}_C$ gives

$$R = \left(\frac{15}{8\pi} \frac{\sigma}{(\rho_Q - \rho_N)^2} \right)^{1/3} \simeq 5.0 \text{ fm} \left(\frac{\sigma}{\sigma_0} \right)^{1/3} \left(\frac{\rho_Q - \rho_N}{\rho_0} \right)^{-2/3}. \quad (5)$$

In the second formula we have introduced the typical quantities $\rho_0 = -e \cdot 0.4 \cdot \text{fm}^{-3}$ and $\sigma_0 = 50$ MeV/fm². (Symmetric nuclear matter in vacuum has a surface tension $\sigma = 1$ MeV/fm² for which (5) gives $R \simeq 4$ fm which agrees with the fact that nuclei like ⁵⁶Fe are the most stable.) The total Coulomb and surface energy per unit

volume is given for small f by

$$\epsilon_{S+C} = f 9 \left(\frac{\pi}{15} \sigma^2 (\rho_Q - \rho_N)^2 \right)^{1/3} \simeq 44 \text{ MeV fm}^{-3} f \left(\frac{\sigma}{\sigma_0} \frac{\rho_Q - \rho_N}{\rho_0} \right)^{2/3}. \quad (6)$$

The result for f close to unity is given by replacing f by $1 - f$. The case when the volumes of quark and nuclear matter are equal, i.e. alternating layers of QM and NM ($f = 1/2$), is considered in ³. Similar length scales but smaller Coulomb and surface energies are found.

Consequently, for $\sigma \simeq 10 \text{ MeV/fm}^2$ we find from Eq. (5) a radius of $R \gtrsim 3.1 \text{ fm}$, whereas $\sigma \simeq 100 \text{ MeV/fm}^2$ gives $R \gtrsim 6.6 \text{ fm}$. As μ_e decreases with increasing density the length scales increase as $\propto \mu_e^{-2/3}$. The corresponding mass number and charge are a few hundreds and up.

4.4. Melting Temperature

The melting temperature of a bcc Coulomb lattice is⁹

$$T_c \simeq \frac{Z^2 e^2}{170a} = \frac{Z^2 e^2}{170R} f^{1/3}, \quad (7)$$

where a is the distance between lattice points. The large numerical factor of 170 reflects the fact that it only takes a fraction of the usual Coulomb energy $Z^2 e^2 / R$ for two atoms to slide by each other in a lattice. T_c is typically some hundreds of MeV - much larger than temperatures inside neutron stars, which are estimated to reach $\sim 10 \text{ MeV}$ in supernovae cool rapidly thereafter. The mixed phase is therefore *frozen solid*. Already at densities $\sim 2n_0$ a lattice of QM droplets form which is only melted for very small f or equivalently long lattice distance. However, when the Debye screening length of electrons or protons³ is shorter than the lattice distance the droplet charge is effectively screened off and a neutral object is formed, which may diffuse around in all directions.

5. Summary and Consequences

Assuming a first order phase transition a mixed phase of quark and nuclear matter is energetically favored for a wide range of equation of states - unless the surface tension is too large, $\sigma \gtrsim 70 \text{ MeV/fm}^2$. Quantitative calculations depends on the equation of states applied but typically already around a few times nuclear saturation density droplets of quark matter appear in a lattice embedded in nuclear matter in a uniform background of electrons. The existence of such a mixed phase may have a number of consequences:

5.1. Glitches

The solidity of the mixed phase affects quake phenomena, which have been invoked to explain observations in pulsar glitches. Some features have been explained by a simple two-component model of a rotating neutron star that gradually

slows down¹⁰ and becomes less deformed which strains the rigid component (original believed to be the lattice in the crust). In this model the lattice suddenly cracks/quakes and changes its structure towards being more spherical. Consequently, its moment of inertia, I_c , is decreased and its rotation and pulsar frequency increased due to angular momentum conservation. Subsequently, the two components slowly relaxates to a common rotational frequency on a timescale of days due to superfluidity of the other component (the neutron liquid). The *healing parameter* $Q = I_c/I_{tot}$ measured in glitches reveals that for the Vela and Crab pulsar about $\sim 3\%$ and $\sim 96\%$ of the moment of inertia is in the rigid component respectively. Previously the crust was assumed to be the only rigid component and so the Vela should be almost all crust. This would require that the Vela is a very light neutron star - much smaller than the observed ones which all are compatible with $\sim 1.4M_\odot$. If we by the lattice component include not only the the solid crust but also the protons in NM (which is locked to the crust due to magnetic fields) and the solid QM mixed phase

$$I_c = I_{crust} + I_p + I_{QM}, \quad (8)$$

we can better explain the large I_c for the Crab. The moment of inertia of the mixed phase is sensitive to the EoS's used. For example, for a quadratic NM EoS³ increasing the Bag constant from 95 to 110 MeV/fm³ reduces I_c/I_{total} from $\sim 70\%$ to $\sim 20\%$ for a $1.4M_\odot$ neutron star. The structures in the mixed phase would exhibit anisotropic elastic properties, being rigid to some shear strains but not others in much the same way as liquid crystals. Therefore the whole mixed phase might not be rigid.

Furthermore, the energy released in glitches every few years are too large to be stored in the crust only. The recurrence time for giant quakes, t_c , is inversely proportional to the strain energy¹², which again is proportional to the lattice density and the Coulomb energy

$$t_c^{-1} \propto \frac{1}{a^3} \frac{Z^2 e^2}{a}. \quad (9)$$

Since the lattice distance is smaller for the quark matter droplets and their charge larger than for atoms in the crust, the recurrence time is shorter in better agreement with measurements of giant glitches.

So far this is all just speculation and other models as e.g. superfluid vortices pinned to the crust¹³ have been invoked to explain glitches. Detecting core and crust quakes separately or other signs of three components in glitches, indicating the existence of a crust, superfluid neutrons and a solid core, would support the idea of the mixed quark and nuclear matter mixed phase. However, magnetic field attenuation is expected to be small in neutron stars and therefore magnetic fields penetrate through the core. Thus the crust and core lattices as well as the proton liquid should be strongly coupled and glitch simultaneously.

5.2. Cooling

Neutrino generation, and hence cooling of neutron stars could be influenced by the mixed phase. This could come about because nuclear matter in the droplet phase has a higher proton concentration than bulk neutral nuclear matter and this could make it easier to attain the threshold condition for the nucleon direct Urca process⁶. Another is that the presence of the spatial structure of the droplet phase might allow processes to occur which would be forbidden in a translationally invariant system. Also the mere presence of quark matter can lead to fast cooling¹⁴ when $\alpha_s \neq 0$. All these mechanisms lead to faster cooling.

5.3. *Maximum mass and Rotational Speed of Neutron Stars*

The EoS is softened by the phase transition to QM which in both strange stars⁷ and hybrid stars¹ leads to lighter maximum mass neutron stars. The mixed phase has an even softer EoS as that of the double tangent construction in hybrid stars and has therefore a slightly lighter maximum mass¹¹. In all cases, however, the maximum mass depends strongly on the EoS of nuclear and quark matter.

The maximum rotation rate and damping of radial density oscillations¹⁵ depend on bulk and shear viscosities. These in turn depend on the structures inside the mixed phase. As discussed above the viscosities can be enormous for a rigid lattice but might entirely vanish in plate-like structures that may behave as a liquid crystal.

5.4. *Supernovae*

In a supernova the core collapse is stopped by the incompressibility of nuclear matter. The softer the equation of state the denser the matter is compressed before it bounces and the deeper into the gravitational well the star has fallen. Also a softer EoS creates a more coherent shock wave that excites the matter less. Consequently, more gravitational energy is available and can be transferred to neutrino generation which is believed to power the supernova explosion. Besides softening the EoS it was mentioned above that the mixed phase occurred through a first order phase transition. Thus latent heat can be stored which may also affect supernovae.

Due to neutrino trapping and non-zero temperatures the situation is, however, somewhat different in supernovae core collapse than in old neutron stars. In particular the high density of neutrinos in β -equilibrium increase the energy densities and pressures so that typically only around twice nuclear matter densities are reached in cores of supernovae. Thus the amount of quark matter in the newly formed neutron star will be less if any, and the supernova explosion will be less affected as well.

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